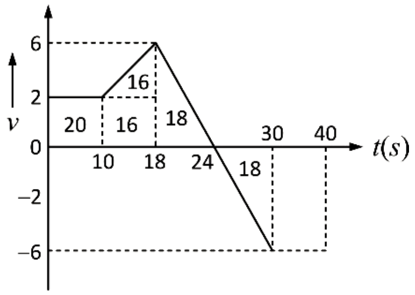
Solutions to Mock JEE MAIN - 3 (CBT) |JEE - 2024

Physics

SINGLE CHOICE

1.(C) At $t = 18\ s$ particle is at $= -16 + (20 + 16 + 16) = 36\ m$



2.(A) $V = \frac{\omega}{k} = \frac{450}{9} = 50\, m/s$; $V = \sqrt{\frac{T}{\mu}}$; So, $T = \mu v^2 = 5\times10^{-3}\times2500 = 12.5\,N$

3.(C) $Q = \Delta U + W$

$\Rightarrow \quad \Delta U = Q - W = 100 - 120 = -20J$ (Negative)

$\Rightarrow \quad \Delta T \rightarrow$ Negative

Temperature will decrease.

From, $PV = nRT \quad \Rightarrow$ decrease

Since, Volume increased as W_{gas} is positive, so pressure will decrease.

Statement (I) is false.

4.(A) $U_i = \frac{1}{2}CV^2 + \frac{1}{2}CV^2 = CV^2$

After opening the switch, charge on disconnected capacitor will remain same whereas potential difference across connected (with battery) capacitor will remain same.

$$\therefore \quad U_f = \frac{1}{2}(KC)V^2 + \frac{(CV)^2}{2KC} = \frac{CV^2}{2}\left(K + \frac{1}{K}\right) = \frac{CV^2}{2}\left(3 + \frac{1}{3}\right) = \frac{10}{6}CV^2$$

$$\therefore \quad \frac{U_i}{U_f} = \frac{CV^2}{\frac{10}{6}CV^2} = \frac{3}{5}$$

5.(B) $KE = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2KE}{m}} \Rightarrow \lambda = \frac{h}{mv} = \frac{h}{\sqrt{2(KE)m}} \Rightarrow \lambda\,\alpha\,\frac{1}{\sqrt{KE}}$

$\Rightarrow \quad \lambda_1 \propto \frac{1}{\sqrt{E_0}}$ & $\lambda_2 \propto \frac{1}{\sqrt{2E_0}} \quad \Rightarrow \quad \frac{\lambda_1}{\lambda_2} = \sqrt{2}$

6.(C) About the fixed point, $MV_0\frac{L}{2} = \left\{\frac{ML^2}{12} + M\left(\frac{L}{2}\right)^2\right\}\omega \quad \Rightarrow \quad \omega = \frac{3V_0}{2L}$

7.(C) **(A)** $C = \frac{Q}{m\cdot\Delta T} \equiv \frac{\left[ML^2T^{-2}\right]}{[M][\theta]} = \left[L^2T^{-2}\theta^{-1}\right]$

(B) $\frac{\Delta Q}{\Delta t} = \frac{KA(\Delta T)}{L} \quad \Rightarrow \quad K = \frac{\Delta Q}{\Delta t}\cdot\frac{L}{A.\Delta T} \equiv \frac{[ML^2T^{-2}][L]}{[T][L^2][\theta]} = [MLT^{-3}\theta^{-1}]$

(C) $k = \dfrac{R}{N} \equiv [ML^2T^{-2}\theta^{-1}]$

(D) $P = \sigma AT^4 \quad \Rightarrow \quad \sigma = \dfrac{P}{AT^4} \equiv \dfrac{[ML^2T^{-3}]}{[L^2][\theta^4]} = [MT^{-3}\theta^{-4}]$

8.(C) In ferromagnetic material, all the molecular magnetic dipoles are pointed in the same direction. When the ferromagnetic material heated, all magnetic dipoles are distributed and get disoriented. Due to which the net magnetic dipole moment becomes very less, so that they behave as paramagnetic material. So, statement I is true.

9.(B) As per standard notations and convention, symbols are (2) & (3) respectively.

10.(B) Condition for no deflection is $eE = Bev$.

$$\Rightarrow \qquad v = \frac{E}{B} = \frac{V}{d \times B} = \frac{3000}{2 \times 10^{-2} \times 2.5 \times 10^{-3}} = 6 \times 10^7\, ms^{-1}$$

If the electron is accelerated through a potential difference V', then $eV' = \dfrac{1}{2}mv^2$

or $\qquad v = \sqrt{\dfrac{2eV'}{m}}$ or $V = \sqrt{2 \times \dfrac{e}{m} \times 10000}$ or $(6 \times 10^7)^2 = 2 \cdot \dfrac{e}{m} \times 10000$

$$\Rightarrow \qquad \frac{e}{m} = \frac{36 \times 10^{14}}{2 \times 10000} = 1.8 \times 10^{11}\, C\,kg^{-1}$$

11.(A) $P_{atm} + P_{gh} = 3P_{atm}$

$h = 20\,m$

Now, velocity of efflux $= \sqrt{2gh} = 20\,m/s$

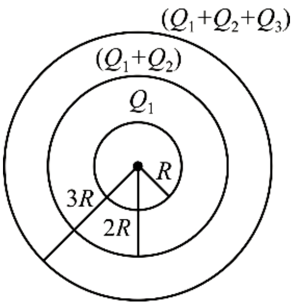
12.(B) $T = mw^2 l$

Stress $= \dfrac{T}{A} = \dfrac{mw^2 l}{A} \le \sigma \quad \Rightarrow \quad \sigma = 4.8 \times 10^7\, N/m^2 \quad \Rightarrow \quad w \le \sqrt{\dfrac{\sigma A}{m\ell}} \quad \Rightarrow \quad w \le 4\, rad/s$

13.(A) $\lambda_m T = \text{constant} \quad \Rightarrow \quad \lambda_0 T_0 = \lambda_1 T_1 \quad \Rightarrow \quad T_1 = \dfrac{4}{3}T_0$

$\therefore \qquad$ Power radiated $\propto T^4 \quad \therefore \quad \dfrac{P_1}{P_0} = \left(\dfrac{4}{3}\right)^4 = \dfrac{256}{81}$

14.(B)

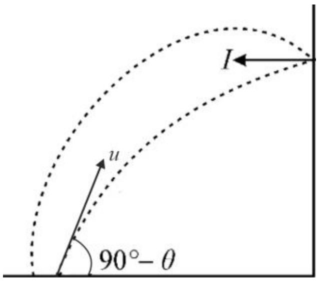


Charge distribution on the outer surface is

So, $\qquad \dfrac{Q_1}{4\pi R^2} = \dfrac{Q_2 + Q_3}{4\pi(2R^2)} = \dfrac{Q_1 + Q_2 + Q_3}{4\pi(3R^2)} \quad \Rightarrow \quad Q_1 : Q_2 : Q_3 = 1 : 3 : 5$

15.(B) As there is no impulse along vertical, there will not be any change in vertical velocity.

Hence, $T = \frac{2u_y}{g} = 2u\frac{\cos\theta}{g}$

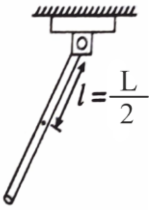


16.(D) $\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)} \quad \Rightarrow \quad \sqrt{2} = \frac{\sin\left(\frac{60°+\delta_m}{2}\right)}{\sin\left(\frac{60°}{2}\right)}$

$\Rightarrow \quad \sin\left(30° + \frac{\delta_m}{2}\right) = \sqrt{2} \times \frac{1}{2} = \frac{1}{\sqrt{2}} \quad \Rightarrow \quad 30° + \frac{\delta_m}{2} = 45° \quad \Rightarrow \quad \delta_m = 30°$

17.(C) $T_1 = 2\pi\sqrt{\frac{L}{g}}$

$T_2 = 2\pi\sqrt{\frac{I}{Mgl}}$



(physical pendulum)

$= 2\pi\sqrt{\frac{\frac{1}{3}ML^2}{Mg\frac{L}{2}}} = 2\pi\sqrt{\frac{2L}{3g}} \quad \therefore \quad \frac{T_1}{T_2} = \sqrt{\frac{3}{2}}$

18.(A) Total energy = – Kinetic energy = – E
i.e., Energy will have to be supplied to escape the electron to infinity = E

19.(D) $V_{rms} = \sqrt{\frac{3RT}{M}}$

$V_{f,\,rms} = 2V_{i,\,rms} \quad \Rightarrow \quad T_f = 4T_i$

For Adiabatic Process : $T_iV_i^{\gamma-1} = T_fV_f^{\gamma-1}$

$\Rightarrow \quad \frac{V_i}{V_f} = \left(\frac{T_f}{T_i}\right)^{\frac{1}{\gamma-1}} = (4)^{1/\left(\frac{5}{3}-1\right)} = (4)^{3/2} = 8$

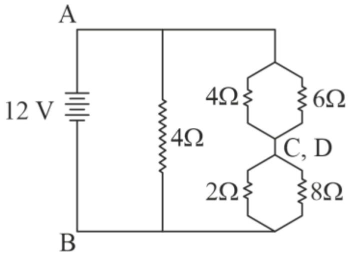
20.(D) Density $\rho = \frac{m}{\pi r^2 L}$

$\therefore \quad \frac{\Delta\rho}{\rho}\times 100 = \left(\frac{\Delta m}{m} + 2\frac{\Delta r}{r} + \frac{\Delta L}{L}\right)\times 100$

After substituting the values, we get the maximum percentage error in density = 4%

NUMERICAL TYPE

1.(6) Equivalent Circuit will be:



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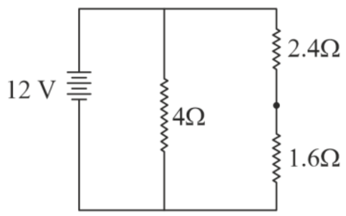

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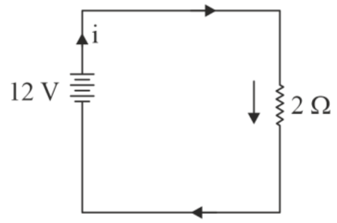
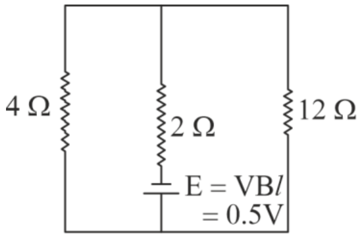


$i = \frac{12}{2} = 6A$

2.(1)



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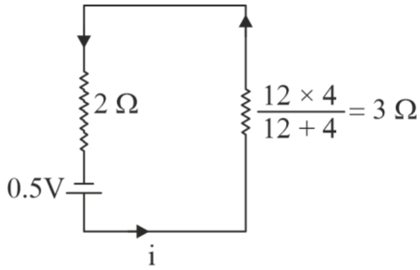


Induced motional emf,

$E = vBl = 4 \times 0.5 \times 0.25 = 0.5 volt$

$i = \frac{E}{R_{eqv.}} = \frac{0.5}{(2+3)} = 0.1\text{A} = 1 \times 10^{-1} A$

3.(1) Applying work kinetic energy theorem from instant block strikes the spring to instant of maximum compression.

$\Delta K.E. = w_{\text{spring}} + w_{\text{friction}}$

$\frac{1}{2} \times 50(2)^2 = \frac{1}{2} k x_m^2 + \mu mg\, x_m$

$100 = \frac{1}{2} \times 100 x_m^2 + 0.1 \times 50 \times 10\, x_m$

$2 = x_m^2 + x_m \quad \Rightarrow \quad x_m = 1m$

4.(2) Change in pressure, $\Delta P = P_1 + P_2$

$\frac{4T}{r} = \frac{4T}{r_1} + \frac{4T}{r_2} \quad \Rightarrow \quad \frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2} \quad \Rightarrow \quad \frac{1}{r} = \frac{1}{3} + \frac{1}{6} = \frac{2+1}{6} = \frac{1}{2} \quad \Rightarrow \quad r = 2$

5.(40) According to the principle of calorimetry, heat lost by hot body = heat gained by cold body

$M \times L_V + M \times C_w \times \Delta T = 200 \times L_f + 200 \times C_w \times \Delta T'$

$M \times 540 + M \times 1 \times (100 - 40) = 200 \times 80 + 200 \times 1 \times (40 - 0)$

$(540 + 60)M = 16000 + 8000$ or $M = 40\ g$

6.(5) Given, translational kinetic energy of $N_2 =$ kinetic energy of electron,

$\frac{3}{2} k_B T = eV \quad \Rightarrow \quad \frac{3}{2} \times 1.38 \times 10^{-23} \times T = 1.6 \times 10^{-19} \times 0.1$

$\Rightarrow \quad T = \frac{2 \times 1.6 \times 10^{-20}}{3 \times 1.38 \times 10^{-23}}$ or $T = 773K = (773 - 273)°C \quad \therefore \quad T = 500°C$

7.(50) For Ammeter : $i_{\max} = i_{G,\,\max}\left(\frac{R_G + S}{S}\right)$

$$\Rightarrow \qquad i_{G,\,\max} = \frac{i_{\max}}{\left(\frac{R_G + S}{S}\right)} = \frac{250}{\left(\frac{R_G + 5}{5}\right)} mA \qquad \ldots\ldots (1)$$

For Voltmeter : $V_{\max} = i_{G,\max}\left(R_G + R_h\right)$

$$\Rightarrow \qquad i_{G,\max} = \frac{V_{\max}}{R_G + R_h} = \frac{25\times10^3}{R_G + 1050} mA \qquad \ldots\ldots (2)$$

From (1) & (2)

$$\frac{250}{\frac{(R_G + 5)}{5}} = \frac{25\times10^3}{(R_G + 1050)}$$

$$\Rightarrow \qquad R_G + 1050 = (R_G + 5)\times\frac{25\times10^3}{250\times5} = (R_G + 5)\times 20 \qquad \Rightarrow \quad R_G = \frac{950}{19} = 50\,\Omega$$

8.(2) In electromagnetic wave, electric field and magnetic field are related as,

$E_0 = B_0 c$

$$\therefore \qquad B_0 = \frac{E_0}{c} = \frac{6}{3\times10^8} = 2\times10^{-8} T$$

9.(60) Power output $= \frac{\text{Energy output}}{\text{Time}} = \frac{\text{Number of atm}\times\text{Energy per atm}}{\text{Time}}$

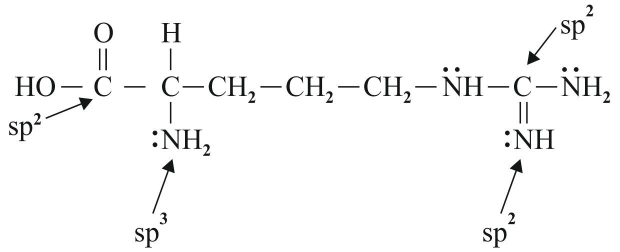
$$= \frac{\left(6\times10^{23}\times\frac{864}{235}\right)\times(235\times10^6\times1.6\times10^{-19})}{16\times86400\,s} = 60\,MW$$

10.(3) $a_{c.m.} = \frac{mg\sin\theta}{\left(m + \frac{I}{R^2}\right)} = \frac{mg\sin\theta}{\left(m + \frac{mR^2/2}{R^2}\right)} = \frac{2}{3} g\sin\theta = 3$

Chemistry

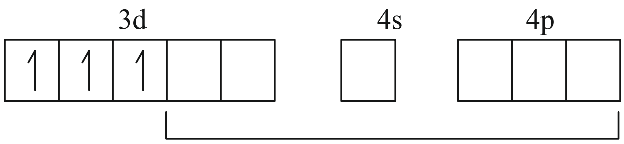
SINGLE CHOICE

1.(C)



2.(A)

Cr^{3+} : $[Ar]\ 3d^3$



d^2sp^3

Inner orbital complex — Octahedral

Paramagnetic

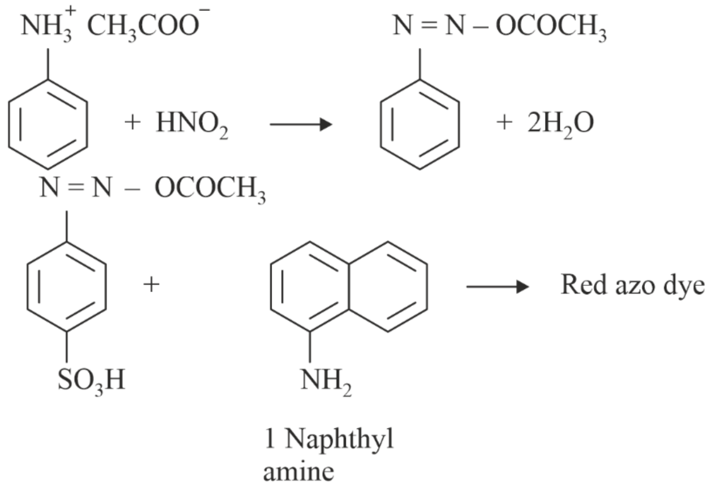
3.(C) Statement I is true but Statement II is false

During the reaction between $KMnO_4$ & oxalic acid, Mn^{2+} acts as an autocatalyst.

4.(C) **(A)** I_2 + starch $\rightarrow$ Blue complex

(B)

$$NO_2^- + CH_3COOH \longrightarrow HNO_2 + CH_3COO^-$$



(C) $S^{2-} + Na_2[Fe(CN)_5NO] \rightarrow Na_4[Fe(CN)_5NOS]$

Purple/violet complex

(D) $SO_4^{2-} \xrightarrow{(CH_3COO)_2Pb} \underset{\text{white}}{PbSO_4(s)}$

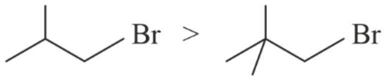
5.(A) Conc. H_2SO_4 is not used during the preparation of Mohr's salt as it can oxidize Fe^{2+} to Fe^{3+}.

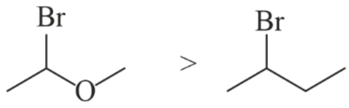
6.(B) Statement 1 and 3 are incorrect.

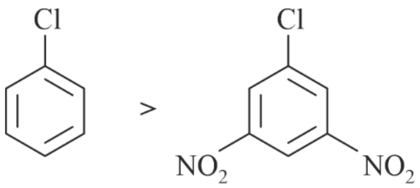
(A) Mo(VI) and W(VI) are more stable than Cr(VI)

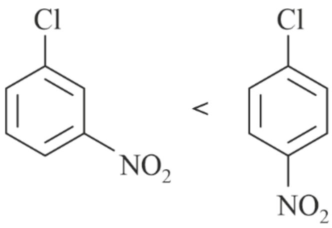
(C) Cr^{2+} is stronger reducing agent than Fe^{2+}.

7.(D) Glycogen is stored in the liver of animals.

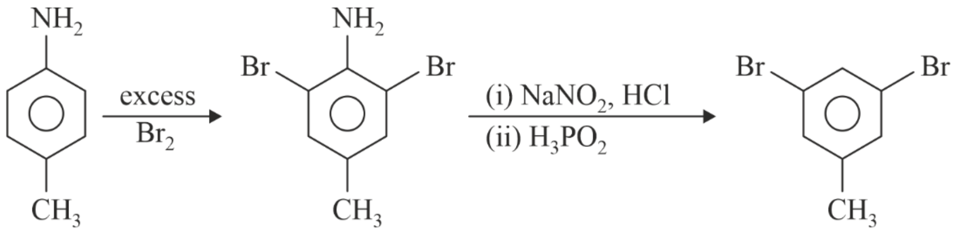
8.(A) **(1)** S_N2 

(2) S_N1 

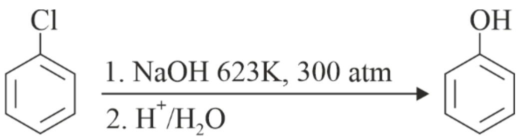
(3) Electrophilic substitution 

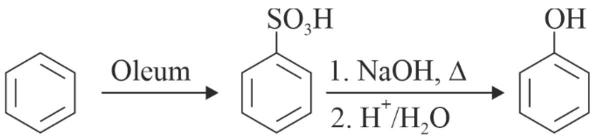
(4) Nucleophilic substitution 

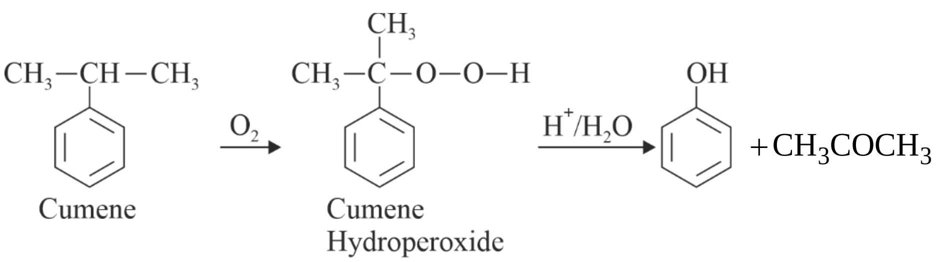
9.(B)

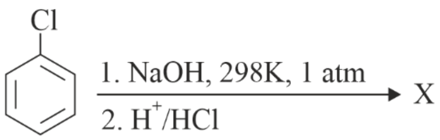


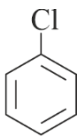
10.(B)

11.(B) **I.** 

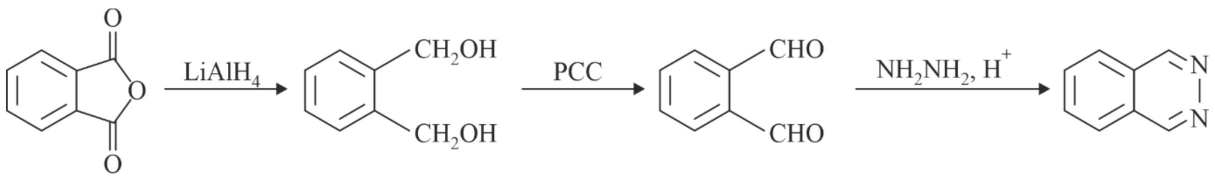
II. 

III. 

IV. 

 doesn't get hydrolyzed by NaOH at 298K.

12.(D)



13.(C) $\frac{1}{4}P_4 + \frac{6}{4}Cl_2 \to PCl_3(\ell)$ $\quad -\frac{1270}{4}$

$PCl_3(\ell) + Cl_2 \to PCl_5(s)$ $\quad -137$

Adding, $\frac{1}{4}P_4 + \frac{5}{2}Cl_2 \to PCl_5$ $\quad$ –454.5 kJ/mol

14.(D) Mass of $CO_2 = 330\,g$

Number of moles of $CO_2 = \frac{330}{44} = 7.5\,\text{moles}$

$n_{CO_2} = n_C = 7.5\,\text{moles}$

mass of carbon = Number of moles $\times$ molar mass $= 7.5\,\text{mol} \times 12\,\text{g mol}^{-1}$

Mass of carbon = 90g

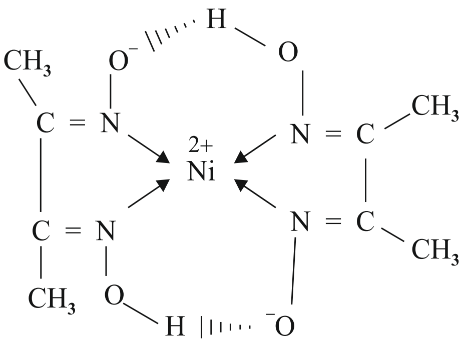
% of carbon $= \frac{\text{Mass of carbon} \times 100}{\text{Mass of organic compound}}$

% of carbon $= \frac{90}{120} \times 100 = 75\%$

% of hydrogen = 100 – % of carbon $= 100 - 75 = 25\%$

15.(A)

Complex is



red color

X = 4; Y = 2

16.(B) A $\to$ Q, B $\to$ R, C $\to$ S, D $\to$ P

17.(B) $\underset{\text{phosphoric acid}}{H_3PO_4} \to$ Tribasic acid

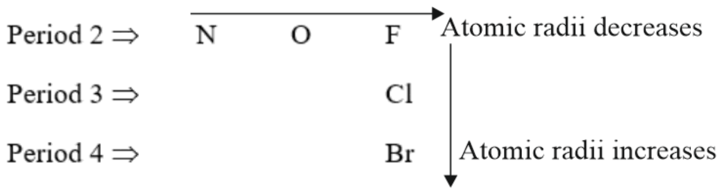
$\underset{\text{Ammonium hydroxide}}{NH_4OH} \to$ Monoacidic base

$\underset{\text{Water}}{H_2O} + H^+ \rightleftarrows \underset{\text{Conjugate acid}}{H_3O^+}$

18.(C) Statement 1 is true and statement 2 is false

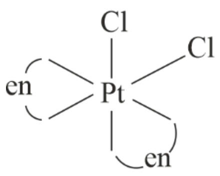
19.(B) Samarium belongs to 4f series. Its electronic configuration is [Xe] $4f^6\ 6s^2$

20.(B) The decreasing order of the atomic radii of N, O, F, Cl, and Br is Br > Cl > N > O > F.

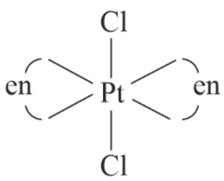


NUMERICAL VALUE TYPE

1.(2)



Cis – $[Pt(en)_2Cl_2]^{2+}$

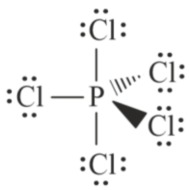


trans – $[Pt(en)_2Cl_2]^{2+}$

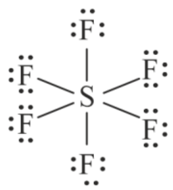
2.(11)

	Bond Order
N_2	3.0
N_2^+	2.5
N_2^-	2.5
N_2^{2-}	2.0
He_2	0
Li_2	1.0
Sum	**11**

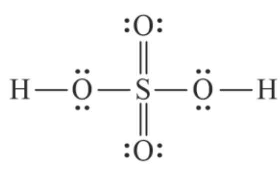
3.(75)



PF_5
10 electrons around the P atom



SF_6
12 electrons around the S atom



H_2SO_4
12 electrons around the S atom

X = 34

Y = 15 + 18 + 8 = 41

4.(3) $kt = 2.303 \log \frac{[A]_0}{[A]_t}$

$$\log\left(\frac{[A]_0}{[A]_t}\right) = \frac{5.25 \times 10^{-5} \times 3.66 \times 60 \times 60}{2.303} = 0.3$$

$$\frac{[A]_0}{[A]_t} = 2 \quad \Rightarrow \quad [A]_t = 0.5\,M$$

$$r = 5.25\times10^{-5}\times0.5 = 2.625\times10^{-5}\ \text{mol L}^{-1}\,\text{s}^{-1}$$

5.(37) $Fe^{+2} + 2e^- \rightarrow Fe$; –0.44 V

$Fe^{+3} + e^- \rightarrow Fe^{+2}$; 0.77 V

$Fe^{+3} + 3e^- \rightarrow Fe$

$$E° = \frac{-0.44\times2+0.77}{3} = -0.0366 \approx -36.66\,mV$$

$$E^{\circ}_{Fe/Fe^{3+}} = 36.66 = 37\,mV$$

6.(6)

$$Cu \rightarrow [Ar]4s^1\,3d^{10}$$

$$\downarrow -e^-$$

$$Cu^+ \rightarrow [Ar]4s^0\ \ \underset{n=3,l=2}{\underset{\downarrow}{3d^{10}}}$$

$x = 2$; $y = 3 \Rightarrow x\times y = 3\times2 = 6$

7.(3)

Acidic Oxides	Amphoteric Oxides	Basic Oxides	Neutal Oxides
$\left.\begin{matrix}Cl_2O_7\\ SO_2\\ P_4O_{10}\end{matrix}\right\}(3)\rightarrow Y$	$Al_2O_3\}(1)\rightarrow X$	$\left.\begin{matrix}Na_2O\\ CaO\\ BaO\end{matrix}\right\}(3)$	$\left.\begin{matrix}N_2O\\ NO\end{matrix}\right\}$

$X \times Y = 1 \times 3 = 3$

8.(2) $30 = CRT$

$$C = \frac{30}{0.0821\times298} = 1.23$$

$$\Delta T_f = 1.86\times1.23 = 2.2878 \quad \Rightarrow \quad -x = -2.2878$$

9.(4)

(1) $O_2(g, 10\,atm) \rightarrow O_2(g, 2\,atm)$: $\Delta S > 0$

(2) $O_2(g, 273\,K) \rightarrow O_2(g, 300\,K)$: $\Delta S > 0$

(3) $Cl_2(g) \rightarrow 2Cl(g)$: $\Delta S > 0$

(4) $C(diamond) \rightarrow C(graphite)$: $\Delta S > 0$

(5) $N_2(g, 2\,atm) \rightarrow N_2(g, 4\,atm)$: $\Delta S < 0$

(6) $C_2H_4(g) + H_2(g) \rightarrow C_2H_6(g)$: $\Delta S < 0$

10.(10) $N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g) : K_C = 10$

$$\Delta n_{(g)} = (2) - (1+3) = -2$$

$$K_P = K_C(RT)^{\Delta n_{(g)}} = 10\times(0.08\times400)^{-2}$$

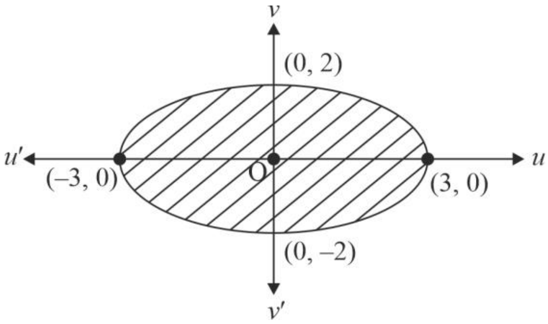
$$K_P = \frac{10}{(0.08\times400)^2} \quad \Rightarrow \quad K_P = 9.76\times10^{-3}$$

Mathematics

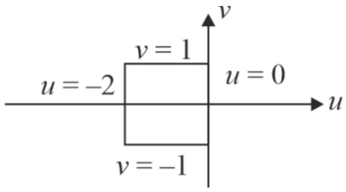
SINGLE CHOICE

1.(C) Let's effect a change of origin $a - 6 = u$ and $b - 5 = v$.

The set B becomes $\frac{u^2}{9} + \frac{v^2}{4} \le 1$ which is the interior of the ellipse.



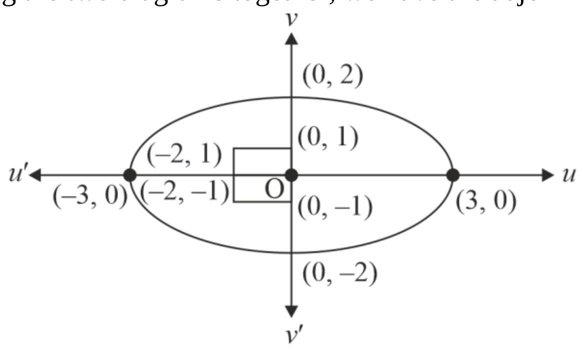
Now the set A becomes



$|u + 1| < 1$ i.e., $-2 < u < 0$

$|v| < 1$ i.e., $-1 < v < 1$, which is a square.

Drawing the two diagrams together, we have the adjoining figure.



So, $A \subset B$ as all the four points (0, 1), (0, –1), (–2, –1) and (–2, 1) lie inside the ellipse.

2.(B) Given, $f(x - y) = f(x)\,f(y) - f(a - x)\,f(a + y)$

Let $x = 0 = y$

Then, $f(0) = (f(0))^2 - (f(a))^2$

$\Rightarrow \quad 1 = 1 - (f(a))^2 \quad \Rightarrow \quad f(a) = 0$

$\therefore \quad f(2a - x) = f(a - (x - a)) = f(a)\,f(x - a) - f(a + x - a)\,f(0)$

$\Rightarrow \quad f(2a - x) = 0 - f(x)(1) = -f(x) \quad (\because f(a) = 0, f(0) = 1)$

3.(B) $\bar{a} = (1+\lambda)\hat{i} + (1-\lambda)\hat{j} + 2\hat{k}$

$\bar{a} \cdot \hat{j} = |\bar{a}| \cos 60°$

$1 - \lambda = \sqrt{(1+\lambda)^2 + (1-\lambda)^2 + 2^2} \cdot \frac{1}{2}$

$2(1 - \lambda) = \sqrt{2 + 2\lambda^2 + 4}$ ……(i)

$\sqrt{2}(1-\lambda) = \sqrt{\lambda^2 + 3}$

$2(1-\lambda)^2 = \lambda^2 + 3$

$2(1+\lambda^2 - 2\lambda) = \lambda^2 + 3$

$\lambda^2 - 4\lambda - 1 = 0$

$\lambda = \dfrac{4 \pm \sqrt{20}}{2} = 2 \pm \sqrt{5}$

But since angle is acute with +ve y-axis, so $1-\lambda > 0$ in (i)

$\therefore \quad \lambda = 2 - \sqrt{5}$

4.(C) As p, q, and r are roots of $x^3 - ax^2 + bx - c = 0$, we can make:

$$x^3 - ax^2 + bx - c = (x-p)(x-q)(x-r) \qquad \ldots\ldots \text{(i)}$$

Area of triangle with sides p, q and r $= \sqrt{s(s-p)(s-q)(s-r)}$ where $s = \dfrac{p+q+r}{2} = \dfrac{a}{2}$

$\Rightarrow \quad \text{Area} = \sqrt{\dfrac{a}{2}\left(\dfrac{a}{2}-p\right)\left(\dfrac{a}{2}-q\right)\left(\dfrac{a}{2}-r\right)} \qquad \ldots\ldots \text{(ii)}$

In identify (i) put $x = \dfrac{a}{2}$ to get:

$$\left(\frac{a}{2}-p\right)\left(\frac{a}{2}-q\right)\left(\frac{a}{2}-r\right) = \left(\frac{a}{2}\right)^3 - a\left(\frac{a}{2}\right)^2 + b\left(\frac{a}{2}\right) - c \quad \Rightarrow \quad \text{Area } = \frac{1}{4}\left[a(4ab - a^3 - 8c)\right]^{1/2}$$

5.(C) $({}^{21}C_1 - {}^{10}C_1) + ({}^{21}C_2 - {}^{10}C_2) + \ldots. + ({}^{21}C_{10} - {}^{10}C_{10})$

$= ({}^{21}C_1 + {}^{21}C_2 + \ldots + {}^{21}C_{10}) - ({}^{10}C_1 + {}^{10}C_2 + \ldots + {}^{10}C_{10}) = (2^{20} - 1) - (2^{10} - 1) = 2^{20} - 2^{10}$

6.(C) Given, $\omega = \dfrac{1 + (1-8\alpha)z}{1-z}$

For ω to be purely imaginary, $\omega + \overline{\omega} = 0$

i.e., $\dfrac{1+(1-8\alpha)z}{1-z} + \dfrac{1+(1-8\alpha)\overline{z}}{1-\overline{z}} = 0$

$\Rightarrow \quad [1+(1-8\alpha)z][1-\overline{z}] + [1+(1-8\alpha)\overline{z}][1-z] = 0$

$\Rightarrow \quad [1-\overline{z} + (1-8\alpha)z - (1-8\alpha)z\overline{z}] + [1 - z + (1-8\alpha)\overline{z} - (1-8\alpha)z\overline{z}] = 0$

$\Rightarrow \quad 2 - (z+\overline{z}) + (1-8\alpha)(z+\overline{z}) - 2(1-8\alpha) = 0 \qquad (\because z\overline{z} = |z|^2 = 1)$

$\Rightarrow \quad 2 - (z+\overline{z}) + (z+\overline{z}) - 8\alpha(z+\overline{z}) - 2 + 16\alpha = 0$

$\Rightarrow \quad 16\alpha = 8\alpha(z+\overline{z})$

Either $z + \overline{z} = 2$ or $\alpha = 0$

But $z+\overline{z} = 2$ is not possible $\quad [\because \text{Re}(z) \neq 1]$

$\therefore \quad \alpha = 0$

7.(C) We have, $\omega = \dfrac{5+3z}{5(1-z)}$

$\Rightarrow \quad 5\omega(1-z) = 5 + 3z \quad \Rightarrow \quad 5\omega - 5\omega z = 5 + 3z \quad \Rightarrow \quad z = \dfrac{5\omega - 5}{5\omega + 3}$

Now, $|z| < 1 \quad$ [Given]

$\Rightarrow \quad \left|\dfrac{5\omega-5}{5\omega+3}\right| < 1 \quad \Rightarrow \quad 5|\omega - 1| < |5\omega + 3| \quad \Rightarrow \quad |\omega - 1| < \left|\omega + \dfrac{3}{5}\right|$

$\Rightarrow \quad |x+iy-1| < \left|x+iy+\frac{3}{5}\right|$ [where $\omega = x+iy$]

$\Rightarrow \quad (x-1)^2 + y^2 < \left(x+\frac{3}{5}\right)^2 + y^2 \quad \Rightarrow \quad x^2+1-2x < x^2+\frac{9}{25}+\frac{6}{5}x$

$\Rightarrow \quad \left(2+\frac{6}{5}\right)x > 1-\frac{9}{25} \quad \Rightarrow \quad x > \frac{1}{5} \quad \therefore \quad \text{Re}(\omega) > \frac{1}{5}$

8.(A) Given, $x^2\sin\theta - x(\sin\theta\cos\theta+1)+\cos\theta = 0$

$D = (1+\sin\theta\cos\theta)^2 - 4\sin\theta\cos\theta = (1-\sin\theta\cos\theta)^2$

$\therefore \quad x = \dfrac{1+\sin\theta\cos\theta \pm (1-\sin\theta\cos\theta)}{2\sin\theta} \quad \Rightarrow \quad x = \cos\theta$ or $\text{cosec}\,\theta$

$\because \quad 0<\theta<45^\circ$ & $\alpha<\beta \quad \Rightarrow \quad \alpha = \cos\theta$ and $\beta = \text{cosec}\,\theta$

Now, $\displaystyle\sum_{n=0}^{\infty}\left(\alpha^n + \left(-\frac{1}{\beta}\right)^n\right) = \sum_{n=0}^{\infty}(\cos\theta)^n + \sum_{n=0}^{\infty}(-\sin\theta)^n = \frac{1}{1-\cos\theta}+\frac{1}{1+\sin\theta}$

9.(B) *M*, *EEE*, *D*, *I*, *T*, *RR*, *AA*, *NN*

R _ _ *E*

Two empty places can be filled with identical letters (*EE*, *AA*, *NN*] in 3 ways.

Two empty places can be filled with distinct letters (*M*, *E*, *D*, *I*, *T*, *R*, *A*, *N*] in ${}^8P_2$ ways.

$\therefore$ Number of words formed = 3 + ${}^8P_2$ = 59

10.(D) Clearly *OP* = *OQ* = 1 unit and $\angle POQ = \alpha - 2\theta$ where $\angle POX = \theta$.

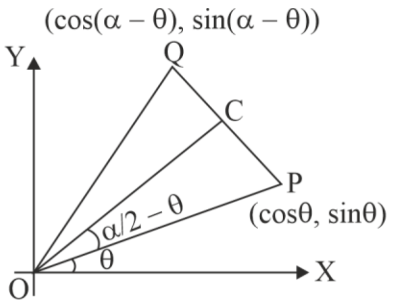
As ΔOPQ is isosceles triangle.

Draw perpendicular from *O* to the line *PQ*, *C* is the foot of the perpendicular and is the midpoint of *PQ*.

$\therefore$ *Q* is the reflection of *P* in the line *OC*,

$\therefore \quad \angle COP + \angle POX = \dfrac{\alpha}{2}$

$\therefore$ Slope of *OC* = $\tan\left(\dfrac{\alpha}{2}\right)$.



11.(C) $b = \displaystyle\lim_{x\to 0}\frac{f\left(2\sin^2\frac{x}{2}\right)}{g(x)\left(2\sin^2\frac{x}{2}\right)^2}\times\frac{\left(\sin\frac{x}{2}\right)^4}{\left(\sin^2\frac{x}{2}\right)\left(\cos^2\frac{x}{2}\right)}$

$b = \displaystyle\lim_{x\to 0}\frac{a}{g(x)}\times\tan^2\frac{x}{2} \qquad \left[\because \lim_{x\to 0}\frac{f(x)}{x^2} = a\right]$

$$\Rightarrow \quad \lim_{x\to 0}\frac{x^2}{g(x)}=\frac{4b}{a}$$

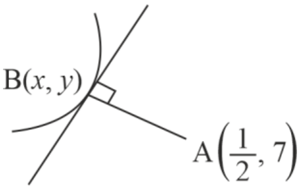
Then, $\lim_{x\to 0}\frac{g(1-\cos 2x)}{x^4}=\lim_{x\to 0}\frac{g(2\sin^2 x)}{(2\sin^2 x)^2}\times\frac{(2\sin^2 x)^2}{x^4}=\frac{a}{b}$

12.(B) Given curve is, $y-x^{3/2}=7$ …(i)

$$\Rightarrow \quad y=7+x^{3/2} \quad\Rightarrow\quad \frac{dy}{dx}=\frac{3}{2}x^{1/2}$$

Distance will be minimum when AB is perpendicular to the curve.

$$\therefore \quad \left(\frac{3}{2}\sqrt{x}\right)\left(\frac{7-y}{\frac{1}{2}-x}\right)=-1$$



$$\Rightarrow \quad \frac{3}{2}\sqrt{x}\left(\frac{-x^{3/2}}{\frac{1}{2}-x}\right)=-1 \qquad \text{[Using (i)]}$$

$$\Rightarrow \quad 3x^2+2x-1=0 \;\Rightarrow\; (3x-1)(x+1)=0 \;\Rightarrow\; x=\frac{1}{3} \qquad [\because x\ge 0\text{, so neglecting } x=-1]$$

From (i), $y=7+\left(\frac{1}{3}\right)^{3/2}$

Now, $AB=\sqrt{\left(\frac{1}{2}-\frac{1}{3}\right)^2+\left(7-7-\left(\frac{1}{3}\right)^{3/2}\right)^2}=\frac{1}{6}\sqrt{\frac{7}{3}}$

13.(A) Angle bisector of OA and OB where O is the origin is given by

$$x-y=0 \qquad \text{…… (i)}$$

Since, the distance of C from (i) is $\frac{3}{\sqrt{2}}$

$$\therefore \quad \frac{|\beta-(1-\beta)|}{\sqrt{2}}=\frac{3}{\sqrt{2}} \;\Rightarrow\; |2\beta-1|=3 \;\Rightarrow\; \beta=2 \text{ or } -1$$

14.(D) We have, $A'=[1\quad 1\quad 1]$

$$A'B=[1\quad 1\quad 1]\begin{bmatrix}9^2 & -10^2 & 11^2\\ 12^2 & 13^2 & -14^2\\ -15^2 & 16^2 & 17^2\end{bmatrix}=\left[9^2+12^2-15^2 \quad -10^2+13^2+16^2 \quad 11^2-14^2+17^2\right]$$

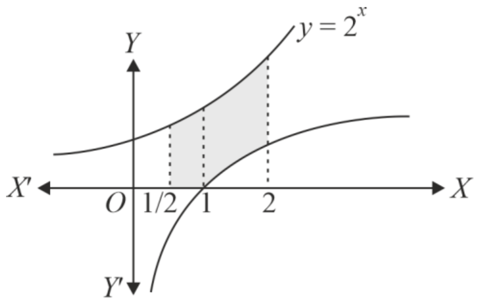
$$A'BA=\left[9^2+12^2-15^2 \quad -10^2+13^2+16^2 \quad 11^2-14^2+17^2\right]\begin{bmatrix}1\\1\\1\end{bmatrix}$$

$$=\left[9^2+12^2-15^2-10^2+13^2+16^2+11^2-14^2+17^2\right]=[539]$$

15.(A) For $\frac{1}{2} \le x \le 1$, we have $0 \le y \le 2^x$ and for $1 \le x \le 2$, we have $\log_e x \le y \le 2^x$

Required area = Area of shaded region

$$= \int_{1/2}^{1} 2^x dx + \int_{1}^{2} (2^x - \log_e x) dx = \left.\frac{2^x}{\log_e 2}\right|_{1/2}^{1} + \left[\frac{2^x}{\log_e 2} - (x \cdot \log_e x - x)\right]_1^2$$



$$= \frac{2^1}{\log_e 2} - \frac{2^{1/2}}{\log_e 2} + \left\{\frac{2^2}{\log_e 2} - 2 \cdot \log_e 2 + 2\right\} - \left\{\frac{2}{\log_e 2} + 1\right\}$$

$$= (\log_e 2)^{-1}[2 - 2^{1/2} + 2^2 - 2] - 2\log_e 2 + 2 - 1$$

$\Rightarrow \quad \alpha = 4 - \sqrt{2}, \beta = -2$ and $\gamma = 1$

Now, $(\alpha + \beta - 2\gamma)^2 = (4 - \sqrt{2} - 2 - 2)^2 = (-\sqrt{2})^2 = 2$

16.(A) The given differential equation is $\frac{\cos x}{2 + \sin x} dx + \frac{dy}{y+1} = 0$

$\Rightarrow \quad \ln(2 + \sin x) + \ln(y + 1) = \ln c$

$\Rightarrow \quad (y + 1)(2 + \sin x) = c \quad \Rightarrow \quad 2 \times 2 = c \Rightarrow c = 4$

Thus, $y + 1 = \frac{4}{2 + \sin x} \quad \Rightarrow \quad y = \frac{2 - \sin x}{2 + \sin x} \quad \Rightarrow \quad y\left(\frac{\pi}{2}\right) = \frac{1}{3}$

17.(A) $f(x) = \sin x + \sin x \int_{-\pi/2}^{\pi/2} f(t) dt + \cos x \int_{-\pi/2}^{\pi/2} t f(t) dt$

$\Rightarrow \quad f(x) = \sin x \left[1 + \int_{-\pi/2}^{\pi/2} f(t) dt\right] + \cos x \int_{-\pi/2}^{\pi/2} t f(t) dt$

$\Rightarrow \quad f(x) = a \sin x + b \cos x \quad \ldots\ldots \text{(i)}$

Where $a = 1 + \int_{-\pi/2}^{\pi/2} (a \sin t + b \cos t) dt$ [Using (i)]

$a = 1 + [-a \cos t + b \sin t]_{-\pi/2}^{\pi/2} = 1 + 2b \quad \ldots\ldots \text{(ii)}$

and $b = \int_{-\pi/2}^{\pi/2} (at \sin t + bt \cos t) dt$ [Using (i)]

$b = [-at \cos t + a \sin t + bt \sin t + b \cos t]_{-\pi/2}^{\pi/2} = 2a \quad \ldots\ldots \text{(iii)}$

Solve (ii) and (iii) to get : $a = -\frac{1}{3}, b = -\frac{2}{3}$

a and b max $[f(x)] = \sqrt{a^2 + b^2} = \frac{\sqrt{5}}{3}$; min $[f(x)] = -\sqrt{a^2 + b^2} = -\frac{\sqrt{5}}{3}$

18.(A) Consider, $\lim_{x\to 0}\left(\frac{1+f(3+x)-f(3)}{1+f(2-x)-f(2)}\right)^{1/x}$ (1^{∞} form)

$$=\lim_{x\to 0}\left(1+\frac{f(3+x)-f(3)-f(2-x)+f(2)}{1+f(2-x)-f(2)}\right)^{1/x}=e^{\lim_{x\to 0}\frac{f(3+x)-f(3)-f(2-x)+f(2)}{x[1+f(2-x)-f(2)]}}$$

$$=e^{\lim_{x\to 0}\frac{f(3+x)-f(3)}{x\cdot 1}}\cdot e^{\lim_{x\to 0}\frac{f(2)-f(2-x)}{x\cdot 1}}=e^{f'(3)}\cdot e^{f'(2)}=e^{f'(3)+f'(2)}$$

$$=e^{0}=1 \qquad [\because f'(3)+f'(2)=0]$$

19.(D) Here, $e_1=\sqrt{1-\frac{4}{18}}=\sqrt{\frac{7}{9}}$ and $e_2=\sqrt{1+\frac{4}{9}}=\sqrt{\frac{13}{9}}$

Since, (e_1, e_2) is a point on the ellipse $15x^2+3y^2=k$

$\therefore \qquad 15e_1^2+3e_2^2=k \;\Rightarrow\; k=15\left(\frac{7}{9}\right)+3\left(\frac{13}{9}\right) \;\Rightarrow\; k=16$

20.(C) We have, $x^2+x\sin\theta-2\sin\theta=0$

Since α, β are the roots of the given quadratic equation.

$\therefore \qquad \alpha+\beta=-\sin\theta$ and $\alpha\beta=-2\sin\theta$ ……(i)

Now, $\frac{\alpha^{12}+\beta^{12}}{\left(\frac{1}{\alpha^{12}}+\frac{1}{\beta^{12}}\right)(\alpha-\beta)^{24}}=\frac{\alpha^{12}\beta^{12}(\alpha^{12}+\beta^{12})}{(\alpha^{12}+\beta^{12})(\alpha-\beta)^{24}}=\frac{\alpha^{12}\beta^{12}}{(\alpha-\beta)^{24}}$

$$=\frac{(\alpha\beta)^{12}}{[(\alpha+\beta)^2-4\alpha\beta]^{12}}=\left[\frac{\alpha\beta}{(\alpha+\beta)^2-4\alpha\beta}\right]^{12}=\left[\frac{-2\sin\theta}{\sin^2\theta+8\sin\theta}\right]^{12}=\frac{2^{12}}{(8+\sin\theta)^{12}}$$

NUMERICAL VALUE TYPE

1.(32) Three digits numbers formed with digits 1, 2, 3, 4, 5 which are divisible by 3 = 4 × 3! = 24

Three digits numbers formed with digits 1, 2, 3, 4, 5 which are divisible by 5 = 4 × 3 = 12

Three digits numbers formed with digits 1, 2, 3,4,5 which are divisible by 15 = 1 × 2 × 2! = 4

$\therefore$ Required numbers = 24 + 12 – 4 = 32

2.(12) $\frac{S_m}{S_n}=\frac{S_{5n}}{S_n}=\frac{\frac{5n}{2}(6+(5n-1)d)}{\frac{n}{2}(6+(n-1)d)}=\frac{5((6-d)+5nd)}{(6-d)+nd}$

For $d=6$, we have S_{5n}/S_n, independent of 'n'. Also, for $d=0$, it's trivially true.

So, $a_2=9$ or 3.

3.(2021)

Clearly, $c_2=a_2+b_2=a_1-3+2b_1=12$ (Given)

$\Rightarrow \qquad a_1+2b_1=15$ …… (i)

and $c_3=a_3+b_3=a_1-6+4b_1=13$ (Given)

$\Rightarrow \qquad a_1+4b_1=19$ …… (ii)

On solving (i) and (ii), we get $b_1=2$ and $a_1=11$

Now, $\sum_{k=1}^{10}c_k=\sum_{k=1}^{10}a_k+\sum_{k=1}^{10}b_k$

Where $\sum_{k=1}^{10} a_k = \frac{10}{2}(2a_1 + (10-1)d) = \frac{10}{2} \times (22 + 9 \times (-3)) = 5 \times (22 - 27) = -25$

and $\sum_{k=1}^{10} b_k = \frac{b_1(r^{10}-1)}{(2-1)} = \frac{2 \times (2^{10}-1)}{2-1}$

$2 \times (2^{10}-1) = 2046 \Rightarrow \sum_{k=1}^{10} c_k = 2021$

4.(11) Probability of hitting the target, $p = \frac{1}{2}$

Probability of non-hitting the target, $q = 1 - \frac{1}{2} = \frac{1}{2}$

Let n bombs are required to destroy the target. According to question, we have

$1 - ({}^nC_0 p^0 q^n + {}^nC_1 p^1 q^{n-1}) \ge \frac{99}{100}$

$\Rightarrow \quad 1 - \frac{1}{2^n} - \frac{n}{2^n} \ge \frac{99}{100} \quad \Rightarrow \quad \frac{1}{100} \ge \frac{n+1}{2^n} \quad \Rightarrow \quad 2^n \ge (n+1)100$

At $n = 10$, $2^{10} \ge 1100$; not possible

At $n = 11$, $2^{11} \ge 1200$ which is true

5.(5) $\lim_{x \to \frac{\pi}{4}} \frac{(\cos x + \sin x)^3 - 2\sqrt{2}}{1 - \sin 2x} = \lim_{x \to \frac{\pi}{4}} 2\sqrt{2} \frac{\left(\cos^3\left(x - \frac{\pi}{4}\right) - 1\right)}{1 - \sin 2x}$

$$= -2\sqrt{2} \lim_{x \to \frac{\pi}{4}} \frac{\left(1 - \cos\left(x - \frac{\pi}{4}\right)\right)}{1 - \cos\left(\frac{\pi}{2} - 2x\right)} \lim_{x \to \frac{\pi}{4}} \left[1 + \cos^2\left(x - \frac{\pi}{4}\right) + \cos\left(x - \frac{\pi}{4}\right)\right]$$

$$= -6\sqrt{2} \lim_{x \to \frac{\pi}{4}} \frac{2\sin^2\left(\frac{\frac{\pi}{4} - x}{2}\right)}{2\sin^2\left(\frac{\pi}{4} - x\right)}$$

$$= -6\sqrt{2} \lim_{x \to \frac{\pi}{4}} \frac{\sin^2\left(\frac{\frac{\pi}{4} - x}{2}\right)}{4\sin^2\left(\frac{\frac{\pi}{4} - x}{2}\right)\cos^2\left(\frac{\frac{\pi}{4} - x}{2}\right)} = -\frac{6\sqrt{2}}{4} = -\frac{3\sqrt{2}}{2}$$

$\therefore \quad P = 3$ & $Q = 2$

6.(36) Two circles touches each other if $C_1C_2 = |r_1 \pm r_2|$

Here, $C_1(3, 0)$ and $C_2(0, 4)$. So, $C_1C_2 = 5$

Also, $r_1 = \sqrt{9 + 0 - 8} = 1$ and $r_2 = \sqrt{0 + 16 - 16 + k} = \sqrt{k}$

$\therefore \quad \sqrt{k} + 1 = 5$ or $\left|\sqrt{k} - 1\right| = 5 \quad \Rightarrow \quad k = 16$ or $k = 36 \quad \Rightarrow$ Maximum value of k is 36.

7.(2) $\lim\limits_{x\to 1}\dfrac{f(x)-1}{g(x)} = \lim\limits_{x\to 1}\dfrac{\log_2\left(1+\tan\left(\frac{\pi x}{4}\right)\right)-1}{x-1}$

$$= \lim_{x\to 1}\frac{\frac{1}{\log_e 2}\cdot\frac{1}{1+\tan\left(\frac{\pi x}{4}\right)}\cdot\sec^2\left(\frac{\pi x}{4}\right)\cdot\frac{\pi}{4}}{1} \qquad \left(\because \frac{0}{0}\text{form, } \therefore \text{ using L - H rule}\right)$$

$$= \frac{1}{\log_e 2}\cdot\frac{1}{1+1}\cdot 2\cdot\frac{\pi}{4} = \frac{\pi}{4}\cdot\log_2 e$$

8.(1) $I = \displaystyle\int_0^1 \frac{dx}{1+\sqrt{x}+\sqrt{1+x}} = \int_0^1 \frac{2t\,dt}{1+t+\sqrt{1+t^2}}$ (Put $x = t^2$)

$$= \int_0^1 \frac{2t\left(1+t-\sqrt{1+t^2}\right)}{(1+t)^2-\left(1+t^2\right)}dt = \int_0^1 (1+t-\sqrt{1+t^2})\,dt$$

$$= t+\frac{t^2}{2}-\left[\frac{t}{2}\sqrt{1+t^2}+\frac{1}{2}\ell n(t+\sqrt{1+t^2})\right]\Bigg|_0^1 = 1+\frac{1}{2}-\frac{1}{\sqrt{2}}-\frac{1}{2}\ell n(1+\sqrt{2})-0$$

So, $I = \dfrac{3}{2}-\dfrac{1}{\sqrt{2}}-\dfrac{1}{2}\ell n(1+\sqrt{2})$

9.(80) Given, $(1-y)^m(1+y)^n$

$= 1 + a_1y + a_2y^2 + \ldots\ldots + a_{m+n}y^{m+n}$ ……(i)

and $a_1 = a_2 = 10$

Differentiating (i) both sides with respect to y, we get

$-m(1-y)^{m-1}\times(1+y)^n + (1-y)^m\cdot n(y+1)^{n-1} = a_1 + 2a_2y + 3a_3y^2 + \ldots\ldots + a_{m+n}(m+n)y^{m+n-1}$

$\Rightarrow$ $n(y+1)^{n-1}(1-y)^m - m(1-y)^{m-1}(1+y)^n$

$= a_1 + 2a_2y + 3a_3y^2 + 4a_4y^3 + \ldots\ldots$ ……(ii)

Again, differentiating (ii) w.r.t. y, we get

$n(n-1)(y+1)^{n-2}\cdot(1-y)^m - n(y+1)^{n-1}\cdot m(1-y)^{m-1}$

$+m(m-1)\cdot(1-y)^{m-2}\cdot(1+y)^n - m(1-y)^{m-1}\cdot n(1+y)^{n-1}$

$= 2a_2 + 6a_3y + 12a_4y^2 + \ldots\ldots$ ……(iii)

Putting y = 0 in (ii) and (iii), we get

$n - m = 10$ ……(iv)

and $m^2 + n^2 - (m+n) - 2mn = 20$

$\Rightarrow$ $(n-m)^2 - (m+n) = 20$ ……(v)

By (iv) and (v), we get

$(10)^2 - (m+n) = 20 \quad\Rightarrow\quad m+n = 80$

10.(6) The given equation of hyperbola is $\dfrac{x^2}{4}-\dfrac{y^2}{2}=1$

and equation of straight line is $2x - y = 0$.

Equation of tangent at (x_1, y_1) is $\dfrac{x\cdot x_1}{4}-\dfrac{y\cdot y_1}{2}=1$

$\Rightarrow \quad x \cdot x_1 - 2y \cdot y_1 - 4 = 0$ and this is parallel to $2x - y = 0$

$\therefore \quad \dfrac{x_1}{2y_1} = 2 \quad \Rightarrow \quad x_1 - 4y_1 = 0 \qquad \ldots\ldots\text{(i)}$

Now as the point (x_1, y_1) lies on given hyperbola.

$\therefore \quad \dfrac{x_1^2}{4} - \dfrac{y_1^2}{2} - 1 = 0$

On solving (i) and (ii), we get

$y_1 = \pm\sqrt{\dfrac{2}{7}}$ and $x_1 = \pm\sqrt{\dfrac{32}{7}}$

$\therefore \quad x_1^2 + 5y_1^2 = \dfrac{32}{7} + 5 \times \dfrac{2}{7} = \dfrac{32+10}{7} = \dfrac{42}{7} = 6$